

Towards the detecting of pseudo-Hermitian anomalies for negative square masses neutrinos in intensive magnetic fields

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Abstract

One of the primary goals of contemporary physics of neutrinos after discovery of their masses become the investigation of their electromagnetic properties. This is a necessary step for creation of new physics beyond the Standard Model (SM), which no longer can claim the role theory explaining everything phenomenon of the Universe. On this it should draw attention because SM remains consistent local scheme for any value of the masses of particles $0 \leq m < \infty$. Now the masses of elementary particles can exceed even Planck's mass $m_{Planck} \simeq 10^{19} GeV$, which is the largest scale mass in the Universe. For solving this problem of studying of electromagnetic interactions of neutrino we suggest use the methods of relativistic quantum theory with the limiting mass $m \leq M$. The restriction of mass spectrum of fermions can be obtained in the frame of non-Hermitian (pseudo-Hermitian) fermion systems having the direct application to the neutrino physics. The systems of the similar type include so-called parity-time-symmetric (\mathcal{PTS}) - models, which already are used in various fields of modern physics and, in particular for experimental investigation of \mathcal{PTS} -optics. We also hope that experimental problem neutrinos with the negative mass squared could be also solved on the basis of \mathcal{PTS} -notions taking into account the characteristics of neutrinos of new type ("exotic neutrino") emerging in the theory with the Maximal Mass.

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1 Introduction

It is well known the Nobel Prize in Physics was awarded in 2015 jointly to Takaaki Kajita and Arthur B. McDonald "for the discovery of neutrino

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oscillations, which shows that neutrinos have mass". This discovery has completely changed our understanding of the innermost properties of matter and showed that SM cannot be the comprehensive theory of the fundamental constituents of the Universe. However obviously, that of past successes of the SM is so high that new models which is designed for modifying SM, must contain practically all basic principles lying in the basis of already existing theory. In this connection we should to note that main extension SM may be connected with a generalization of notion Hermiticity.

Indeed it is known that one of the fundamental postulates of quantum theory is the requirement of Hermiticity of physical parameters. This condition not only guarantees the reality of the eigenvalues of Hamilton operators, but also implies the preservation in time of the probabilities of the considered quantum processes. However, as it was shown [1], Hermiticity is a sufficient but it is not a necessary condition. It turned out that among non-Hermitian Hamiltonians it is possible to allocate a wide range pseudo-Hermitian Hamiltonians with providing the real energy spectrum and the development of systems over time with preserving unitarity.

Now it is well-known fact, that the reality of the spectrum in models with a non-Hermitian Hamiltonian is a consequence of \mathcal{PT} -invariance of the theory, i.e. a combination of spatial and temporary parity of the total Hamiltonian: $[H, \mathcal{PT}]\psi = 0$. When the \mathcal{PT} symmetry is unbroken, the spectrum of the quantum theory will be real. This surprising results explain the growing interest in this problem which was initiated by Bender and Boettcher's observation [1]. For the past a few years has been studied a lot of new non-Hermitian \mathcal{PT} -invariant systems (see, for example [2]-[12]).

The algebraic non-Hermitian \mathcal{PTS} γ_5 -extension of the Dirac equation was first studied in [4] and further was developed in our works [8]-[12]. But in the geometrical approach to the construction of Quantum Field Theory (QFT) with fundamental mass which was developed by V.G.Kadyshevsky and his colleagues ([13]-[15]), already has contained equations for motion of fermion with γ_5 -mass extension. Besides in the geometrical QFT we can watch the emergence of new particles, which were named "exotic particles" [13]. At first their the emergence, really considered the prerogative of the geometric approach. However now it is clear, that appearance of such non-ordinary particles is the consequence of using of non-Hermitian models [10]-[12].

This type of Hamiltonians includes the so-called \mathcal{PTS} -models, which already used in various fields of modern physics. The most developed in this respect are such which used in the field of \mathcal{PTS} -optics, where for several

years conducted not only theoretical, but also experimental studies [2],[3]. Although ordinary Hermite interpretation does not disputed \mathcal{PTS} -approach may open up new non-trivial properties early well-studied objects. Thus, development of non-Hermitian approach becomes a very fruitful environment for the creation of new physics beyond the SM [8],[9].

In this connection we want to draw attention that candidate for the new quantum theory of fermions is the modified model with pseudo-Hermitian γ_5 -mass extension of the ordinary quantum Dirac theory. This model can be constructed using the simple generalization Dirac's approach to obtaining his known equations of motion fermions. The Hamiltonian form of this new equations explicitly shows that these Hamiltonian are non-Hermitian, but \mathcal{PT} -invariant and pseudo-Hermitian. This condition not only guarantees the reality of the eigenvalues of Hamilton operators, but also implies the preservation in time of the probabilities of the considered quantum processes [12].

In last decade, many cosmological observations suggest the existence of dark matter and dark energy and they offer to consider them as key components of the content of the Universe (see for example [16],[17]). The goal of detecting the nature of dark matter triggered the development a number of new, highly sensitive detectors that can catch extremely weak interactions between dark matter and objects belonging to SM [18]. Nevertheless, in the literature already exist indications of the existence of the candidates to the structure of dark matter, which is required verification. In particular, there are particles, description of which non corresponds Hermitian theories, but arise in the models with preserving \mathcal{PTS} [10]-[12].

In present paper, we develop the algebraic \mathcal{PTS} -model in the intensive magnetic field, in the frame of which early we have obtained the exact solutions for the real eigenvalues of the energy of charged and neutral fermions [8],[9]. It is very important that there are occur violation of \mathcal{PT} -symmetry, depending on the intensity of the magnetic field and the orientation of fermion spin. According to last investigations in this algebraic model also *may be restriction of mass spectrum of fermions* [8]-[12]. This is limit of mass has far-reaching consequences in the form of the appearance in theory of new particles, no existing in the SM. The main difference of these particles reduced to that they do not obey to the ordinary Dirac equations. For them do not known all properties, but they have equal mass with its partners from ordinary particles. The most perspective for investigation are "exotic neutrinos". At present one can speak about a wide spectrum of different type

neutrinos.

It is enough to remember that there are sterile and active neutrinos. Moreover sterile neutrinos may mix with active neutrinos via a matrix mass. Existence masses of neutrino and mixing implies that neutrinos have magnetic moments. In last time one can often meet with an overviews of electromagnetic properties neutrino, (see, for example, [19]). But as it was noted up in this paper " now there is no positive experimental indication in favor existence electromagnetic properties of neutrinos". With it really is hard not to agree because the interactions of ordinary neutrinos with the electromagnetic fields are extremely weak. However if one to suggest using the "exotic neutrinos" the interaction with magnetic field may be really significantly increased thanks to the coefficient which be equal to the ratio of Maximal Mass and mass of neutrinos $k = M/m_\nu$ [8],[9]. Such experiments in our opinion may be very fruitful for creation the new physics beyond the SM. Perhaps that this effects indeed can be observed in terrestrial experiments. In the papers [8],[9] the energy spectra of the fermions was obtained by us as exact analytical solutions of the modified Dirac equation which taken into account with the interaction of anomalous magnetic moments (AMM) of fermions with intense magnetic fields.

Note also that intense magnetic fields exist in the series of space objects. In particular, magnetic field intensity of the order of $10^{12} \div 10^{13}$ Gauss observed near and inside of the pulsars. Here also may be noted recent discovered objects which are known as sources of soft repeating gamma ray bursts and anomalous X-ray pulsars. Magneto-rotational model was proposed for their description and they were named the magnetars. It is suggested that for such objects, the attainable magnetic fields with a strength up to 10^{15} Gauss. This is very important what fraction of magnetars in the General population of neutron stars can reach 10%[20]. In this connection, we note that processes involving neutrinos in the presence of such strong magnetic fields can render a significant impact on processes that can shape evolution of the astrophysical objects. It can be especially noticeable if in a stream of neutrino may be a certain percentage of particles with non-Hermitian characteristics.

On the other hand in 1965 M.A.Markov [21] has proposed hypothesis according to which the mass spectrum of particles should be limited by M.Planck mass $m_{Planck} = \sqrt{\hbar c/G} \approx 10^{19} GeV$. The particles with the limiting mass

$$m \leq m_{Planck} \tag{1}$$

were named by the author the Maximons. However, condition (1) initially was purely phenomenological and until recently it has seemed that this restriction can be applied without connection with SM. And really SM is irreproachable scheme for value of mass from zero till infinity. But in the current situation, however, more and more data are accumulated which bear witness in favor of the necessity of revising notion of Hermiticity. In particular, this is confirmed by abundant evidence that "dark matter", really exists and absorbs a substantial part of the energy density in the Universe.

A new radical approach was offered by V. G. Kadyshevsky and his colleagues [13]-[15], in which the Markov's idea of the existence of a Maximal Mass used as new fundamental principle construction of QFT. This principle refutes the affirmation that mass of the elementary particle can have a value in the interval $0 \leq m < \infty$. In the geometrical theory the condition finiteness of the mass spectrum is postulated in the form

$$m \leq \mathcal{M}, \quad (2)$$

where the Maximal Mass parameter \mathcal{M} , was named by the the *fundamental mass*. This physical parameter is a *new physical constant* along at the speed of light and Planck's constant. The value of \mathcal{M} is considered as a curvature radius of a five dimensional hyperboloid whose surface is a realization of the curved momentum 4-space – the anti de Sitter space. Objects with a mass larger than \mathcal{M} cannot be regarded as elementary particles because no local fields that correspond to them. For a free particle, condition (2) accomplished automatically on surface of a five dimensional hyperboloid. In the approximation $\mathcal{M} \gg m$ the anti de Sitter geometry goes over into the Minkowski geometry in the four dimensional pseudo-Euclidean momentum space ("flat limit")[13].

2 Modified model for the study of non-Hermitian mass parameters

Let us now consider the modified Dirac equations for free massive particles using the γ_5 -factorization of the ordinary Klein-Gordon operator. In this case we will make similar actions as for known Dirac procedure. He particularly has wrote "...need get something like a square root from the operator of Klein-Gordon" [22]. And really if we shall not be restricted to only Hermitian

operators then we can represent the Klein-Gordon operator in the form of a product of two commuting matrix operators with γ_5 -extension of mass:

$$\left(\partial_\mu^2 + m^2\right) = \left(i\partial_\mu\gamma^\mu - m_1 - \gamma_5 m_2\right)\left(-i\partial_\mu\gamma^\mu - m_1 + \gamma_5 m_2\right), \quad (3)$$

where the physical mass of particles m is expressed through the new parameters m_1 and m_2

$$m^2 = m_1^2 - m_2^2. \quad (4)$$

For the function which will obey to the equation of Klein-Gordon

$$\left(\partial_\mu^2 + m^2\right)\tilde{\psi}(x, t) = 0 \quad (5)$$

one can demand that it also satisfies to one of the equations of the first order

$$\left(i\partial_\mu\gamma^\mu - m_1 - \gamma_5 m_2\right)\tilde{\psi}(x, t) = 0; \quad \left(-i\partial_\mu\gamma^\mu - m_1 + \gamma_5 m_2\right)\tilde{\psi}(x, t) = 0 \quad (6)$$

Equations (6) of course, are less common than (5), and although every solution of one of the equations (6) satisfies to (5), reverse approval has not designated. It is also obvious that the Hamiltonians, associated with the equations (6), are non-Hermitian (pseudo-Hermitian), because in them the γ_5 -dependent mass components appear ($H \neq H^+$):

$$H = \vec{\alpha}\mathbf{p} + \beta(m_1 + \gamma_5 m_2) = \vec{\alpha}\mathbf{p} + \beta m e^{\gamma_5 \alpha} \quad (7)$$

and

$$H^+ = \vec{\alpha}\mathbf{p} + \beta(m_1 - \gamma_5 m_2) = \vec{\alpha}\mathbf{p} + \beta m e^{-\gamma_5 \alpha}. \quad (8)$$

Here matrices $\alpha_i = \gamma_0 \cdot \gamma_i$, $\beta = \gamma_0$, $\gamma_5 = -i\gamma_0\gamma_1\gamma_2\gamma_3$ and introduced identical replacement of parameters

$$\sinh(\alpha) = m_2/m; \quad \cosh(\alpha) = m_1/m, \quad (9)$$

where parameter α varies from zero to infinity.

Additionally, an alternative formalism for regarding the systems defined by non-Hermitian Hamiltonians is also known, according to which the spectrum reality for a non-Hermitian system occurs owing to the so-called pseudo-Hermitian properties of a Hamiltonian. A Hamiltonian is called pseudo-Hermitian if it satisfies the condition

$$H^+ = \eta_0 H \eta_0^{-1},$$

where η_0 is a linear Hermitian operator. From (7) and (8) we can find

$$\eta_0 = e^{\gamma_5 \alpha}.$$

It is easy to see from (4) that the mass m , appearing in the equation (5) is real, when the inequality

$$m_1^2 \geq m_2^2, \quad (10)$$

is accomplished. However for variable α which is identical for definitions m_1, m_2 this condition is automatically accomplished in all region $0 \leq \alpha < \infty$.

However this area contains descriptions not only pseudo-Hermitian fermions, which in a result of the Hermitian transition ($m_2 \rightarrow 0, m_1 \rightarrow m$) coincide with the ordinary particles, besides also there are the second region. In this case fermions don't subordinate to the ordinary Dirac equations and for them the Hermitian limit is absent. This is easy to see that if we shall be used for new restrictions of mass parameters the simple expression. Really, if we take into account inequality between arithmetic and geometrical averages of two positive numbers we have

$$m^2 + m_2^2 \geq 2\sqrt{m^2 m_2^2}. \quad (11)$$

Sign of equality takes place when $m = m_2$ and this mass $m = m_2 = M$ has Maximal allowable value for particle mass. Besides of from (4) we can also see that in this point $m_1 = \sqrt{2}M$. Thus value M can be considered as Maximal fermion Mass in pseudo-Hermitian approach under fixed values m_1 and m_2

$$m \leq M. \quad (12)$$

In Markov's terminology [21] one can say that by such a way we defined the mass of Maximon. It is easy to see this mass may be written in the non-Hermitian form

$$m_{Maximon} = \sqrt{2}M + \gamma_5 M. \quad (13)$$

Using also expression (11) one can also obtain that in this task the maximal mass value is expressed by the following way

$$M = m_1^2 / 2m_2. \quad (14)$$

At the Fig.(1a) we can see explicit behavior of the reduced mass distribution of $y(\alpha) = m/M$, depending on parameter α . From this picture follows that

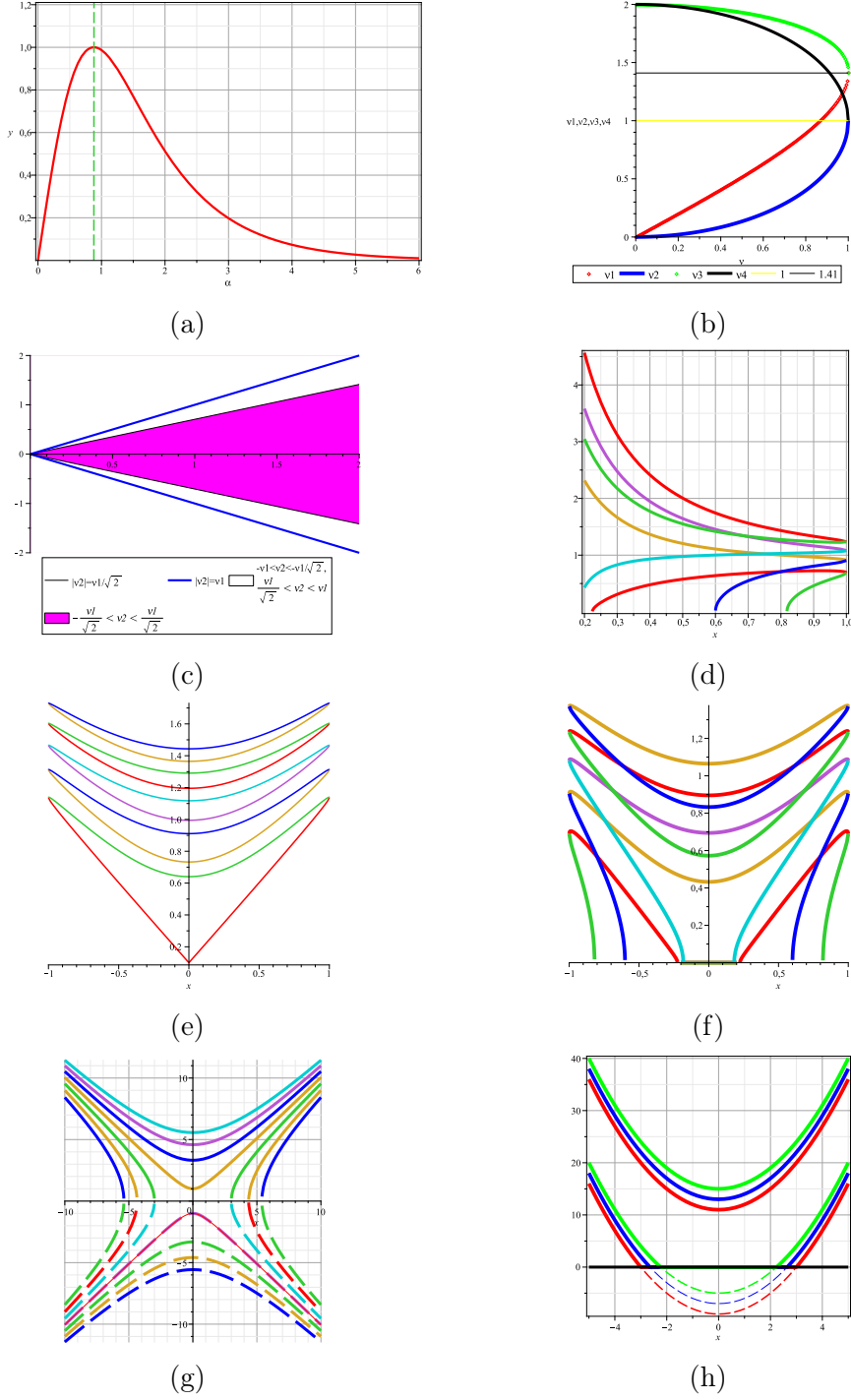


Figure 1: These figure give possibility to verify that restriction in mass spectrum of fermions $m \leq M$ obtained in the framework of non-Hermitian relativistic quantum theory leads to appearance of new particles – the exotic fermions (the plots (a)-(d)). Besides these a number of illustrations (the plots (e)-(h)) bear witness that discovery of exotic neutrinos can indeed open approach to creation of new physics beyond the Standard Model.

the curve, corresponding mass of the considered particles, which has a maximum value of $y(\alpha) = m/M = 1$ in the point $\alpha_0 = 0.881$ and as already noted correspond to Maximon. Till to this value we are dealing with fermions, which have Hermitian limit when $M \rightarrow \infty$ (or $m_2 \rightarrow 0$). But after the value $\alpha_0 = 0.881$, where the maximal value of mass is achieved we deal with decreasing mass of particles. However in this region already no the possibility with the help of the limiting transition to obtain Hermitian mass. Thus in this region particles exist, which in principal differ from the particles of the SM. In particular, massless description fermions may be represented in the form:

$$m_{massless} = 2M(1 + \gamma_5),$$

where M is Maximal Mass of fermion spectrum. Thanks to the presence of a Maximal value of mass parameter we can say about arising of new particles, which no exist in SM.

If the restriction of mass spectrum of elementary particles doesn't exist in Nature then "exotic particles" can't arise. And vice versa, if one can detect their presence it means that limiting mass of fermions really exist.

It is very interesting that early such particles has been observed in geometrical approach to the construction of QFT with fundamental mass [13]-[15]. We believe that exact solutions of modified Dirac-Pauli equations which were obtained by us for the pseudo-Hermitian neutrinos can let valuable information to detect the presence of such "exotic particles". On this indicates also huge increasing of the interaction exotic particles with magnetic fields associated with unusual properties of "exotic neutrinos" wich will be considered in the next section.

3 Motion of Dirac particles, if their own magnetic moment is different from the Bohr's magneton

In this section, we will want touch upon question about description of the motion of new fermions in the magnetic fields, if their own magnetic moment is different from the Bohr's magneton. As it was shown by Schwinger [23] the equation of Dirac particles in the external electromagnetic field A^{ext} taking

into account the radiative corrections may be represented in the form:

$$(\mathcal{P}\gamma - m)\Psi(x) - \int \mathcal{M}(x, y|A^{ext})\Psi(y)dy = 0, \quad (15)$$

where $\mathcal{M}(x, y|A^{ext})$ is the mass operator of the fermion in the external field and $\mathcal{P}_\mu = p_\mu - A^{ext}_\mu$. From equation (15) by means of expansion of the mass operator in a series of according to eA^{ext} with precision not over then linear field terms one can obtain the modified equation. This equation preserves the relativistic covariance and consistent with the phenomenological equation of Pauli obtained in his early papers (see for example [24]).

Now let us consider the model of massive fermions with γ_5 -extension of mass $m \rightarrow m_1 + \gamma_5 m_2$ taking into account the interaction of their charges and AMM with the electromagnetic field $F_{\mu\nu}$:

$$\left(\gamma^\mu \mathcal{P}_\mu - m_1 - \gamma_5 m_2 - \frac{\Delta\mu}{2} \sigma^{\mu\nu} F_{\mu\nu} \right) \tilde{\Psi}(x) = 0, \quad (16)$$

where $\Delta\mu = (\mu - \mu_0) = \mu_0(g - 2)/2$. Here μ - magnetic moment of a fermion, g - fermion gyromagnetic factor, $\mu_0 = |e|/2m$ - the Bohr magneton, $\sigma^{\mu\nu} = i/2(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)$. Thus phenomenological constant $\Delta\mu$, which was introduced by Pauli, is part of the equation and gets the interpretation with the point of view QFT.

The Hamiltonian form of (16) in the homogenies magnetic field is the following

$$i\frac{\partial}{\partial t}\tilde{\Psi}(r, t) = H_{\Delta\mu}\tilde{\Psi}(r, t), \quad (17)$$

where

$$H_{\Delta\mu} = \vec{\alpha}\vec{P} + \beta(m_1 + \gamma_5 m_2) + \Delta\mu\beta(\vec{\sigma}\mathbf{H}). \quad (18)$$

As it was noted earlier a feature of the model with γ_5 -mass contribution is that it may contain another any restrictions of mass parameters in addition to (10). Indeed for the physical mass m may be constructed by infinite number combinations of m_1 and m_2 , satisfying to (4). However besides it need take into account the rules of conformity of this parameters in the Hermitian limit $m_2 \rightarrow 0$. Without this assumption the development of Non-Hermitian models not may be adequate. With this purpose one can determine an additional mass scale M which will depend on m_1, m_2 (see (14)). Indeed this parameter M can put an upper bound on the mass spectrum of particles. Thus fixing

of values m and M as experimentally significant and using (4) and (14) we can obtain expressions defining dependence parameters m_1 and m_2 on them:

$$m_1^\mp = \sqrt{2}M\sqrt{1 \mp \sqrt{1 - m^2/M^2}} \quad (19)$$

$$m_2^\mp = M(1 \mp \sqrt{1 - m^2/M^2}) \quad (20)$$

Using Fig.(1b) we can see values of different branches of $\nu_1^\pm = m_1^\pm/M$ and $\nu_2^\pm = m_2^\pm/M$ as a function of normalized physical parameter $\nu = m/M$. Now the domain of the \mathcal{PT} -symmetry is defined by inequality $0 \leq \nu \leq 1$. For these values of ν the parameters ν_1 and ν_2 correspond to the modified Dirac equation with the Maximal Mass describing the propagation of particles with real masses. But the lower branches ν_1^- , ν_2^- correspond to ordinary particles and upper curves ν_1^+ , ν_2^+ define the exotic partners.

Difference of the ordinary non-Hermitian fermions and exotic particles can understand from the following reasoning. If we accept that Maximal Mass in the fermion spectrum is comparable with Planck mass then using Fig.(1b) it is easy to see, that all known particles of SM are accumulated at the left below corner of this picture. Indeed, according to the estimations which may be produced from (19),(20) using mass of the most massive fermion SM - the mass of top-quark, we have

$$m_{t-quark} = 173,34 \pm 0.76 GeV$$

and for Maximal Mass is equal to Planck's mass

$$M = 10^{19} GeV$$

we can obtain

$$m_1 = m_{t-quark}; \quad m_2 = m_{t-quark}^2/2M = 1.9 \cdot 10^{-15} GeV. \quad (21)$$

And hence for non-Hermitian t-quark one can see

$$m_{t-quark} = m_{t-quark} + \gamma_5 \cdot 2 \cdot 10^{-6} eV. \quad (22)$$

Those we can see that non-Hermitian contribution in this case is extremely small and practically non-observed. In the same time the formulas (19),(20) for the lower signs, i.e. for exotic particles, give values:

$$\tilde{m}_{t-quark} = 2M[1 - m_{t-quark}^2/8M^2] + \gamma_5 2M[1 - m_{t-quark}^2/4M^2]. \quad (23)$$

This means that all information about non-Hermitian exotic fermions now concentrates on upper left corner of the Fig.(1b). And one can wait that most interesting phenomena may be happening with the *the lightest fermions*. Since region of biggest mass $m \sim M$ is still long will be remain unachieved we may assume that this situation resembles the transition from *non-relativistic to the relativistic theory*. As it is known this transition was connected first of all with experimental observations of *limit values of the speed*, which was associated with speed of light. Although the light was always, but to measuring of this value it took much time!

Indeed, if the similar scenario according to the observation of limited value of mass in mass-spectrum of elementary particles will be produced, one should build accelerators till energies 10^{19} GeV and this we must wait even much longer. However on our happiness there is another possibility: searches consequences of the existence of the Maximal Mass value in mass spectrum of elementary particles based on the indirect experimental facts. As the most important in this respect we can specify on the theory of exotic particles, which is developed by us with the help of non-Hermitian (pseudo-Hermitian) approach connected with restrictions of mass spectrum of fermions. In this connection we can say that different regions of undisturbed \mathcal{PTS} ordinary and exotic particles are represented at Fig.(1c). It is easy to see that regions of undisturbed \mathcal{PT} symmetry for these particles are strongly differ.

Performing calculations in many ways reminiscent of similar calculations carried out in the ordinary model in the magnetic field [24], in a result, for modified Dirac-Pauli equations one can find *the exact solution for energy spectrum* [8],[9]:

$$E(\zeta, p_3, p_\perp, \Delta\mu H) = \sqrt{p_3^2 - m_2^2 + \left[\sqrt{m_1^2 + p_\perp^2} + \zeta \Delta\mu H \right]^2} \quad (24)$$

and for eigenvalues of the operator polarization μ_3 we can write in the form

$$k = \sqrt{m_1^2 + p_\perp^2}, \quad (25)$$

where p_3 and p_\perp are longitudinal and transverse components of the neutrinos momentum.

Dependence of of normalized values $E(-1, 0, p_\perp, \Delta\mu H = 0.1)/m$ on the parameter $x = m/M$ for p_3 and the cases $p_\perp = 0, 1, 2, 3, 4$ and $\Delta\mu H = 0.1$. can see at Fig.(1d).

From (24) it follows that in the field where \mathcal{PT} symmetry is unbroken $m \leq M$, all energy levels are real for the case of spin orientation along the magnetic field direction $\zeta = +1$ (see Fig.(1e)).

However, in the opposite case $\zeta = -1$ (fermion spin is oriented against the magnetic field) we have appearance of the imaginary part energy from the ground state of fermion $p_{\perp} = 0$ and other low energy levels, see on Fig.(1f). There are also dependence of $E(-1, 0, p_{\perp}, \Delta\mu H = 0.1)/M$ on the parameter $x = m/M$ for the different values of transverse momentum components p_{\perp}/m : $p_{\perp} = 0, 1, 2, 3, 4$ and $\Delta\mu H = 0.1$. In this case we can see that regions of undisturbed symmetry in the linear approximation of the intensity of the magnetic field H are defined by the following way

$$H \leq \frac{p_0^2}{2\Delta\mu\sqrt{m_1^2 + p_{\perp}^2}}, \quad (26)$$

where $p_0 = \sqrt{m^2 + p_{\perp}^2 + p_3^2}$ - is the ordinary Hermitian energy of the particles.

On the other hand dependence of $E(-1, p_3, p_{\perp}, \Delta\mu H = 0.1)/M$ on the longitudinal components of momentum $x = p_3/M$ for the different values of transverse momentum components p_{\perp}/m : $p_{\perp} = 0, 1, 2, 3, 4$ and $\Delta\mu H = 0.1$. is represented at Fig.(1g). The curves of type of dash are corresponding to the anti-particles. In the case of fermion with spin orientated along the magnetic field $\zeta = +1$ energetic states of particles and anti-particles are separated with the energetic chink order of $2m_{\nu}$. But in the opposite case $\zeta = -1$ we can see that under the interaction of AMM with magnetic field the different branches of particles and anti-particles may be intersected.

It is easy to see that in the case $\Delta\mu = 0$ from (24) one can obtain the ordinary expression for energy of charged particle in the magnetic field *Landau levels*. For this need to note that formula (24) is a valid not only for the neutral fermions, but and for charged particles possessing AMM. In this case one must simply replace the value of transverse momentum of a neutral particle on the quantum value of charged fermions in the magnetic field $p_{\perp}^2 \rightarrow 2\gamma n$, where $\gamma = eH$ and $n = 0, 1, 2, \dots$ modified Landau levels:

$$E(\zeta, p_3, 2\gamma n, \Delta\mu H) = \sqrt{p_3^2 - m_2^2 + \left[\sqrt{m_1^2 + 2\gamma n} + \zeta \Delta\mu H \right]^2} \quad (27)$$

Besides it should be emphasized that from the expression (27), in the Hermitian limit putting $m_2 = 0$ and $m_1 = m$ one can obtain expressions which

was early investigated in [28].

4 Discussions and conclusions

It is well known [29],[30] that in the minimally extended SM the one-loop radiative correction generates neutrino magnetic moment which is proportional to the neutrino mass

$$\mu_{\nu_e} = \frac{3}{8\sqrt{2}\pi^2} |e| G_F m_{\nu_e} = (3 \cdot 10^{-19}) \mu_0 \left(\frac{m_{\nu_e}}{1eV} \right), \quad (28)$$

where G_F -Fermi coupling constant and μ_0 is Bohr magneton. Besides the discussion of problem of measuring the mass of neutrinos (either active or sterile) show that for an active neutrino model we have $\sum m_\nu = 0.320eV$, whereas for a sterile neutrino $\sum m_\nu = 0.06eV$ [31]. However note that the best laboratory upper limit on a neutrino magnetic moment, $\mu \leq 2.910^{-11} \mu_0$, has been obtained by the GEMMA collaboration [32], and the best astrophysical limit is $\mu \leq 3 \cdot 10^{-12} \mu_0$.

The main feature of this approach is that even under the most favorable conditions in respect to the values of the magnetic moments which allow estimate the energy of ordinary neutrinos we must to assume that corresponding intensity of the magnetic fields simply fantastic huge. However for the case of exotic neutrinos we have enormous enhancement of all effects proportionally to the ratio of the Maximal Mass and the neutrinos mass of $k = M/\tilde{m}_\nu$. And because the value of the Maximal Mass M in the SM extends to infinity, the checking of presence of this parameter is advisable start from the upper side of the possible values. For example suitable for this purpose is the value of Planck mass $M = 10^{19}$ GeV. For these parameters the intensity of magnetic fields may be at the level of experimental achievements of contemporary technology even for case of estimation of magnetic moment in according to (28).

Thus using (24) we can write for neutrino energy square

$$E^2(H) = p^2 + \tilde{m}_\nu^2 + 2\zeta \Delta\mu H \sqrt{m_1^2 + p_\perp^2} + (\Delta\mu H)^2. \quad (29)$$

If we consider the case of cold neutrinos, momentum of which is equal to zero, then one could write for neutrino mass square in the magnetic field:

$$\tilde{m}_\nu^2(H) = \tilde{m}_\nu^2 \pm 4\Delta\mu H M. \quad (30)$$

The estimation of parameters $\Delta\mu HM$ with taking into account that Maximal Mass be equal to the Planck's mass and value of magnetic moment value is determined by (28) may be represented in the form:

$$\Delta\mu HM = 10^{-19} \cdot \frac{e}{2m_e} H \cdot 10^{28} \cdot 1eV^2 = \frac{10^{15}}{4} \left(\frac{H}{H_c} \right) 1eV^2, \quad (31)$$

where used that m_e is electron mass and H_c characterizes *quantum magnetic field of electron* $H_c = 4.41 \cdot 10^{13}$ Gauss. Note that field of this strength will be able produce work on the Compton wavelength of electron which be equal to rest energy of the electron. But from (28) for the case of exotic neutrinos we can see that even the values of laboratory magnetic intensity may produced a considerable effects.

One can see that the main progress, is obtained by us in the algebraic way of the construction of the fermion model with γ_5 -mass term is consists of describing of the new energetic scale, which is defined by the parameter $M = m_1^2/2m_2$. This value on the scale of the masses is a point of transition from the ordinary particles $m_2 < M$ to exotic particle $m_2 > M$.

At Fig.(1h) we can see the dependence of squared mass of neutrinos on values longitudinal momentum $x = p_3$ under fixed values of its transverse components p_\perp . Dependence of $E(-1, 0, p_\perp, \Delta\mu H = 0.1)/m$ on the parameter $x = m/M$ for the different values of transverse momentum components p_\perp/m : $p_\perp = 0, 1, 2, 3, 4$ and $\Delta\mu H = 0.1$.

As it was noted, if we suggest that exotic neutrinos have mass (\tilde{m}_ν), which equals to mass of ordinary neutrinos then perhaps, for this type of neutrinos also exist and extremely small anomalous magnetic moments. But thanks to the huge value of Maximal Mass comparable with Planck mass $M \approx 10^{19} GeV$ their interaction with magnetic field may be considerably enhanced ($K = M/\tilde{m}_\nu$). Hence experiments on search of exotic neutrinos can simultaneously answer the question about the existence of the limited spectrum of masses of elementary particles. Therefore if somebody will be able to discover the existence of the exotic neutrinos then it will mean that our World is pseudo-Hermitian Universe and that it has a restriction of mass spectrum of elementary particles.

As it is well known that experiments based tritium to measure the absolute value mass of the neutrino have a long history. The issue of atomic and molecular excitations in tritium, based on neutrino experiments was first raised in the early 1970s [33]. There was able in the first time to set a limit of neutrinos mass about 55-eV [34] and has arisen an understanding about

more lower limits further. The Los Alamos National Laboratory (LANL)-experiment produced an upper limit of $m_\nu < 9.3eV$ at the 95% confidence level [35] was obtained with a 2σ excess. These events was observed in the endpoint of energetic region and quantitatively reported at first time about *a negative values of central points of m_ν^2* . An experiment at Lawrence Livermore National Laboratory (LLNL), also using a gaseous T_2 source, has fixed, which a central value is in good agreement with the LANL result, but with much reduced statistical uncertainties.

In concurrent experiments [37]- [39] has used complex tritium sources. However all of these experiments gave results that were consistent with zero neutrino mass *but with the central values are in regions of the negative-mass squared*. These values were symptomatic and witnessed about an insufficiently successful coordination of theoretical and experimental results for explanation negative dependence of mass squared [40]. Attempts to reduce diapason of such conflicting results furthered interest in molecular-tritium experiments. The limit on the neutrino mass, $m < 2eV$ at an unstated confidence level, is derived from the Mainz [41] and Troitsk [42], [43] experiments, both of which employed a new type of spectrometers. In these experiments a magnetic-adiabatic collimation-with-electrostatic (MAC-E) filter [44] were used. Beta electrons were rotated are in a *region of large magnetic field*.

The Troitsky's experiment, like its predecessors at LANL and LLNL, used a windowless, gaseous tritium source. The gas density and source purity were monitored indirectly by a mass analyzer at the source and by count-rate measurements at a low potential setting. After that an electron gun was run, which mounted upstream of the source. The initial analysis of the data required the inclusion of a step function which added to the spectral shape [42], so-called "Troitsk anomaly." This result, based on a re-analysis of the source calibrations was presented in the form $m < 2.05eV$ at 95% confidence [43]. But at last the final results of Troitsky's experiment, which produces negative square mass in central point of energetic region of neutrinos, can be represented in the form [45]:

$$m_\nu^2 = -0.67 \pm 1.89_{stat} \pm 1.68_{syst} eV^2, \quad (32)$$

at 95% confidence(C.L).

And although in this paper it is noted, that " the result for the mass of neutrino is negative, but the deviation from zero is not statistically significantly". However authors of article [45] also noted that in one of sessions

measurement negative values of square mass neutrinos was obtained the result even

$$m_\nu^2 = -8.06 \pm 6.99 \pm 1.65 eV^2. \quad (33)$$

In this situation we want draw attention to the fact that one can assume we simply deal with experiment measurements of neutrinos mass *with taking into account influence of the magnetic fields*. According to scheme of the the Troitsk experimental installation for measurement of mass the electron antineutrino gas-tritium is injected into a long tube, where a strong magnetic field (up to $H = 0.8T = 8000 Gauss$) is existed. If we suggest that the beam of neutrinos partially consist of pseudo-Hermitian components exotic neutrinos(\tilde{m}) that we can obtain the following evaluation

$$\tilde{m}_\nu^2(H) = \tilde{m}_\nu^2 - 4\Delta\mu HM. \quad (34)$$

Indeed if we consider values, which followed from the ordinary experimental data and using $m_\nu = 1eV$ and formulas (32),(34) with $\tilde{m}_\nu^2(H) = -1eV^2$ one can write

$$4\Delta\mu HM \simeq 2eV^2. \quad (35)$$

Using the last expressions we obtain that observing results follows if the strength of the magnetic fields $H \simeq 8000 Gauss$ [45] and Maximal mass value is equal to $2 \cdot 10^{14} GeV$ when μ_ν taken from (28). However if the values of magnetic moment of neutrinos grows in comparison with (28) then level of the restriction Maximal Mass can be reduced considerably.

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